

**Vedic Mathematics**  
(*Basic Arithmetic*)

*of*  
*Bharati Krishna Tirthaji Maharaj,*  
*(Shankaracarya of Puri Govardhana Muti, Puri)*

by

**Venkatesha Murthy**  
Dean - Mathematics  
International Academy for Creative Teaching  
#52, "IACT Tower", Bellary Road, Hebbal,  
BANGALORE 560 024

Published by  
**National Institute of Vedic Sciences**  
# 58, Raghavendra Colony, Chamaraajpet,  
Bangalore-560058

**Vedic Mathematics**

of Sri Sri Bharati Krishna Tirtha Maharaj  
-by Prof. Venkatesh Murthy,  
Dean, Dept. of Mathematics, IACT, Bangalore.

Published by:  
National Institute of Vedic Sciences  
No.58, Raghavendra Colony,  
Chamaraajpet,  
Bangalore-560018  
Phone: 26914637

Copies: 1000

First Edition : 2009

Copy Right: Authors.

Pages : 52

Price : Rs . 45/-

Printers :  
**Dharshan Prints,**  
No.58, Raghavendra Colony,  
Chamaraajpet,  
Bangalore-560018  
Phone: 26914637, 26607741

# Vedic Mathematics

## (Basic Arithmetic)

of

Bharati Krishna Tirthaji Maharaj

Content	Page #
Foreword	
Preface	
Bharati Krishna Tirthaji Maharaj, A profile.	1
Introduction	3
<b>Chapter I :</b>	
1 (a). The Rule 1 निखिलं नवतः चरमम् दशतः। to find the deviation or complement of a given number from its base.	4
1 (b) Product of two single-digit numbers less than the base 10 applying निखिलम् नवतश्चरमम् दशतः।	5
1 (c) Product of two two-digit numbers nearer to base 100 निखिलम् नवतश्चरमम् दशतः।	6
1 (d) Product of three two-digit numbers nearer to base 1000	7
1 (e) Miscellaneous problems and justification:	8
1 (f) Verification - Vedic Method of Multiplication.	9
2 (a) Product of two single-digit numbers greater than the base 10.	10
2 (b) Product of two two-digit numbers nearer to base 100.	11
2 (c) Product of two three-digit numbers nearer to base 1000.	12
2 (d) Vedic Method of Multiplication - Generalization $(x + a)(x + b) = (x + a + b)x + (ab)$	13

Content	Page #
3 (a) Product of two numbers choosing a working base nearer to these numbers and a multiple of the base.	14
3 (b) Vedic Method of Multiplication - Verification. $(px - a)(px - b) = [p(px - a - b)]x + ab$	15
3 (c) Product of two numbers choosing a working base nearer to these numbers and a multiple of the base.	16
3 (d) Vedic Method of Multiplication - Verification $(px - a)(px + b) = [p(px - a + b) - 1]x + (x - ab)$	18
<b>Chapter II :</b>	
एकोन्युनेन पूर्वेषु । निखिलम् नवतश्चरमम् दशतः।	
1 (a) When the number of nine's in the number having all nines is equal to the number of digits in an other number.	19
1 (b) When the number of nine's in the number having all nines is more than the number of digits in an other number.	20
1 (c) Rationale of the rule for finding the product of a number and another number having all nines	21
1 (d) Geometrical verification of the rule एकोन्युनेन पूर्वेषु । निखिलम् नवतश्चरमम् दशतः। for finding the product of a number and another number having all nines.	22
1 (e) When the number of nine's in the number having all nines is less than the number of digits in an other number: एकोन्युनेन पूर्वेषु । निखिलं नवतः चरमम् दशतः। Special Multiplication facts	23 24

<b>Content</b>	<b>Page #</b>
<b>Chapter III</b> Divisibility test of numbers having unit digit 9 or could be converted so using the rule एकाधिके न पूर्वणा	26
<b>Chapter IV</b> Addition	29
<b>Chapter V</b> Subtraction: The rule निखिलम् नवतश्चरमम् दशतः	31
<b>Reference</b>	35
<b>Answers for a few typical problems given in exercises</b>	36

**Bharati Krishna Tirthaji Maharaj,  
Shankaracarya of Puri Govardhana Mutt, Puri,  
Propounder of Vedic Mathematics - A profile.**

Mathematics is considered the most difficult subject, not only by the ordinary people but by students and the learned as well. At the same time, everyone thinks it is an essential subject to be learnt by all. Many have tried to make it attractive and interesting but their efforts did not have much impact. But there was one divine personality who could make mathematics the simplest, easiest and most attractive of all subjects. Bharati Krishna Tirthaji, who was the Shankaracarya of Puri Govardhana Mutt for 35 long years till his Mahasamadhi in 1960, says that he reconstructed 16 mathematical sutras from materials scattered here and there in the Atharva Veda during 8 years of research and hard work. He wrote 16 volumes and named Vedic Math. But unfortunately, all of them were lost. Despite his failing health and weak eyesight the Swamiji began to rewrite them, but he could complete only one volume, which is presently available. Research work is going on in foreign countries as well as in India to rediscover what was lost in the 16 volumes.

The Swamiji was such a genius that he took M.A. with the highest honours in 8 different subjects from the American College of Science, Rochester, New York at the age of just 20. He contributed learned articles on religion, philosophy, sociology, history, politics, literature *etc.*, while he was a student. In fact, study of the latest researches and discoveries in modern sciences were his hobby till his very last days. At the age of 16, the Madras Sanskrit Association awarded him the title 'Saraswati' for his extraordinary proficiency in Sanskrit.

Bharati Krishna Tirthaji worked with Gopala Krishna Gokhale from 1905 in connection with the National Education Movement and the South African issue. He was chosen the first Principal of the National College at Rajamahendri founded in 1908 by the Nationalists for the propaganda of education based on Indian culture and heritage catering to the needs of an Indian to attain all round development, as distinct from the system introduced by the British. He left that post in 1911 to practice Brahma Sadhana at Sringeri. He was initiated to samnyasa at Benares by the Shankaracarya Sri Trivikrama Thirthaji of Saradapeeth and became Swami Bharathi Krishna Thirthaji. In 1921, he became the Shankaracarya of Saradapeeth. In 1925, he became the Shankaracarya of Puri Govardhana Math. He toured all over India and abroad to propagate Dharma which he defined as "*the sum total of the means necessary for speedily making and permanently keeping all the people, individually as well as collectively superlatively comfortable, prosperous, happy and joyous in all aspects (including the physical, mental, intellectual, educational, economical, social, political, psychic, spiritual etc., ad infinitum)*". He founded Sri Viswa Punarnirmana Sangha at Nagpur in 1953 for the all-round reconstruction of India and through it of the world.

Mathematicians all over the world say that Vedic Mathematics is applicable to all the branches of mathematics and sciences. To propagate Vedic Sciences, the National Institute of Vedic Sciences under the guidance of of Founder and Chief Patron His Holiness Sri Sri Vijnanandhi theertha Swamiji is organizing lectures, symposiums and workshops for teachers, pupils and public along with publications.

## Introduction

## Chapter I

Ten digits used in the International place-value system of numerals for numbers are 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0 and therefore International place-value system of numerals is called base ten system or denary system of place value. The digits from 1 to 9 are named *significant digits/ figurates*. Place-value of each digit in the numeral of a number increases in multiples of 10 from right to left (as we look at it) starting with 1.

For example, the numeral of a number *twenty thousand sixty-five* is 20,065, because its expansion is;

$$20,065 = 2 \times 10000 + 0 \times 1000 + 0 \times 100 + 6 \times 10 + 5 \times 1.$$

Place-values of digits in the numeral 20,065 is increasing in multiples of 10 from right to left (as you look at it). Digit 0 is a special kind of digit. It occupies all the places of place-values where *no figurate/ significant digit* is spelt in a number.

Place-value of *figurate/ significant digit* 5 in 20,065 is 1.

Place-value of *figurate/ significant digit* 6 in 20,065 is 10.

Place-value of the significant digit 2 in 20,065 is 10000.

Since *no figurate/ significant digit* has occupied the place-values of 100 and 1000, their place-values spelt. But numeral of the number *twenty thousand sixty-five* is 20065, but 0 has occupied these place-values (which are not spelt) and for this reason 0 is called the *null figurae* or *null*.

*The place-value of each digit in the numeral of a number moves (increases) towards left (in multiples of ten from the right extreme).*

*This fact is stated as अङ्कानाम् वामतोऽगतिः ।*

Give the value of 7 in the following.

- (i) 837, (ii) 72 (iii) 8796 (iv) 73006 (v) 170008

**I. Rule : निखिलम् नवतश्चरम् दशतः ।**

**(All from nine and last (significant digit) from ten)**

**1(a).** The Rule 1 is helpful to find deviation or complement of a given number from its base.

Base of a single-digit number is 10. Base of numbers from 1 to 9 is 10

Base of a two-digit number is 100. Base of numbers from 10 to 99 is 100.

Base of a three-digit number is 1000. Base of numbers from 100 to 999 is 1000, and so on.

Deviation of a single digit number 7 (from its base) is 3. 3 is the complement of number 7 to its base 10.

Similarly, complement/ deviation of number 857 is 143 (from its base 1000).

**The Rule : निखिलम् नवतश्चरम् दशतः ।** (All from nine and last (significant digit) from ten) is useful to find the deviation of a number from its base.

To find the complement/ deviation of 857 from its base 1000, subtract 8 and 5 from 9, and the last 7 from 10.

Therefore complement/ deviation of 857 (from its base) is  $(1000 - 857) = 143$ .

Complement/ deviation of 807 is 193.

Complement/ Deviation of 8950200 is 1049800. Because, the digits 8, 9, 5 and 0 are subtracted from 9 and the last significant digit 2 is subtracted from 10, and then 2 is followed by zeroes.

**Exercise I. 1 (a):** - Write the complements/ deviation of the following.

- (i) 68 (ii) 502 (iii) 9405 (iv) 89699 (v) 2950 (vi) 990  
(vii) 38730 (viii) 1111 (ix) 5099000 (x) 8888880000

**I. 1(b) Product of two single-digit numbers less than the base 10 applying** निखिलम् नवतश्रमम् दशतः :-

**Example 1:** - Find  $6 \times 8$  base = 10

In numbers 6 and 8, digits 6 and 8 are the last digits also.

Therefore, Deviation of 6 to base  $10 = 6 - 10 = -4$

and deviation of 8 to base  $10 = 8 - 10 = -2$

Write numbers in LHS and their deviations from the base in RHS.

L H S	R H S
(number)	(deviation)
6	-4
8	-2
4	8

Write  $(6 - 2) = (8 - 4) = 4$  in LHS taking numbers crosswise.

Write  $(-4) \times (-2) = 8$  in RHS as shown.

**Then,  $6 \times 8 = 48$**

**Example 2:** - Find  $7 \times 9$  base = 10

In numbers 7 and 9, digits 7 and 9 are the last digits also.

Therefore, Deviation of 7 to base  $10 = 7 - 10 = -3$

and deviation of 9 to base  $10 = 9 - 10 = -1$

Write numbers in LHS and their deviations from the base in RHS.

L H S	R H S
(number)	(deviation)
7	-3
9	-1
6	3

Write  $(7 - 1) = (9 - 3) = 6$  in LHS taking numbers crosswise.

Write  $(-3) \times (-1) = 3$  in RHS as shown.

**Then,  $7 \times 9 = 63$**

**Exercise I. 1(b):** - Find the product of the following using the method shown above: - (i)  $8 \times 6$  (ii)  $8 \times 9$  (iii)  $5 \times 9$  (iv)  $9 \times 9$

(v)  $9 \times 6$  (vi)  $8 \times 8$  (vii)  $7 \times 7$  (viii)  $9 \times 3$  (ix)  $6 \times 8$  (x)  $4 \times$

9

**I. 1(c) Product of two two-digit numbers nearer to base 100 applying** निखिलम् नवतश्रमम् दशतः :-

**Example 1:** - Find  $98 \times 97$  base = 100

Deviation of 98 to base  $100 = 98 - 100 = -02$  and deviation of 97 to base  $100 = 97 - 100 = -3$

Write numbers in LHS and their deviations from the base in RHS.

L H S	R H S
(number)	(deviation)
98	-02
97	-03
95	06

Write  $(98 - 3) = (98 - 2) = 95$  in LHS taking numbers crosswise. Write  $(-3) \times (-2) = 6$  in RHS as shown.

**Then,  $98 \times 97 = 9506$**

**Example 2:** - Multiply 95 and 85 base = 100

Deviation of 95 to base  $100 = 95 - 100 = -05$  and deviation of 85 to base  $100 = 85 - 100 = -15$

Write numbers in LHS and their deviations from the base in RHS.

L H S	R H S
(number)	(deviation)
95	-05
85	-15
80	75

Write  $(95 - 15) = (85 - 5) = 80$  in LHS taking numbers crosswise. Write  $(-05) \times (-15) = 75$  in RHS as shown.

**Then,  $95 \times 85 = 8075$**

**Exercise I. 1(c):** - Find the product of the following using the method shown above; (i)  $94 \times 96$  (ii)  $96 \times 87$  (iii)  $85 \times 75$

6

**I.1(d) Product of three two-digit numbers nearer to base 1000: -**

**Example 1: -** Find  $998 \times 987$  base = 1000

Deviation of 998 to base 1000 =  $998 - 1000 = -002$  and deviation of 987 to base 1000 =  $987 - 1000 = -013$

Write numbers in LHS and their deviations from the base in RHS.

L H S	R H S
(number)	(deviation)
998	-002
987	-013
985	026

Write  $(998 - 013) = (987 - 002) = 985$  in LHS taking numbers crosswise. Write  $(-002) \times (-013) = 026$  in RHS as shown.

**Then,  $998 \times 987 = 985026$**

**Example 2: -** Find  $986 \times 988$  base = 1000

Deviation of 986 to base 1000 =  $986 - 1000 = -014$  and deviation of 988 to base 1000 =  $988 - 1000 = -012$

Write numbers in LHS and their deviations from the base in RHS.

L H S	R H S
Number	Deviation
986	- 014
988	- 012
974	168

LHS:  $(986 - 012)$  or  $(988 - 014) = 974$

RHS:  $(-014) (-012) = 168$

**Exercise I.1(d): -** Find the product of the following using the method shown above.

- (i)  $994 \times 996$  (ii)  $986 \times 990$  (iii)  $995 \times 975$  (iv)  $982 \times 988$
- (v)  $991 \times 979$  (vi)  $983 \times 998$

**I.1(e) Miscellaneous problems and justification: -**

**Example 1: -** Multiplying 99988 and 99984 base = 1,00,000

L H S	R H S
99,988	-00,012
99,984	-00,016
12	00,192

$99988 \times 99984 = 1200192$

because;

Real value of the answer = L H S x base + R H S

$$99988 \times 99984 = 12 \times 1,00,000 + 00,192 = 1200192$$

**Example 2: - A challenging Problem**

Multiplying 99,99,998 and 984 base = 1,00,00,000

L H S	R H S
99,99,998	-00,00,002
00,00,984	-99,99,016
982	1,99,98,032

$$99,99,998 \times 984 = 982,99,98,032 = 9,83,99,98,032$$

because;

Real value of the answer = L H S x base + R H S

$$99,99,998 \times 984 = 982 \times 1,00,00,000 + 1,99,98,032 = 9,82,00,00,000 + 1,99,98,032 = 9,83,99,98,032$$

**Exercise I.1(e): -** Find the product of the following using the method shown above.